2/12/18 "Intelligence plus character, that is the true goal of education."-Martin Luther King Jr.

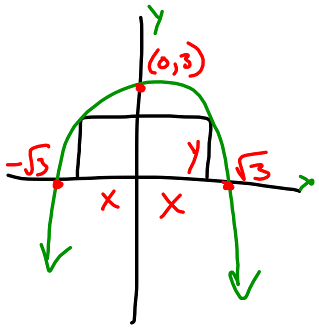
HW: Test 1 on Thursday 2/15

AIM: Optimization Continued

Warm Up:

Take out your notebook. Today's problems are not in the packet.

1. What are the dimensions that maximize the area of a rectangle having two lower corners on the x-axis and the two upper corners on the graph of $y = 3 - x^2$? What is the maximum area?



Restrictions:

$0 < y < 3$

$0 < x < \sqrt{3}$

$y = 3 - 0^2$

$y = 3$

$0 = 3 - x^2$

$x^2 = 3$

$x = \pm\sqrt{3}$

Area

$A = 2xy$

Secondary

$y = 3 - x^2$

$A = 2x(3 - x^2)$

$A = 6x - 2x^3$

$A' = 6 - 6x^2$

$0 = 6 - 6x^2$

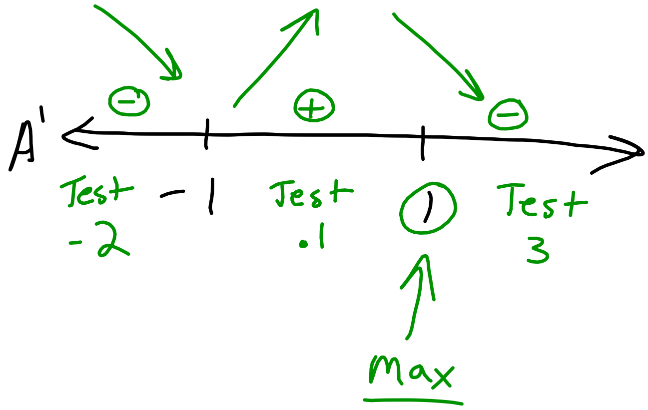
$\frac{6x^2}{6} = \frac{6}{6}$

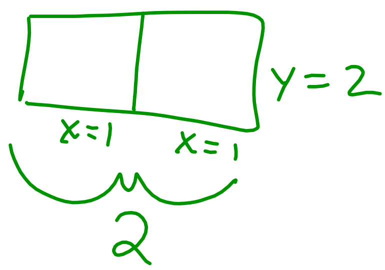
$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

reject -1 b/c of restriction





To find y:

$y = 3 - (1)^2$

$y = 2$

Dimensions

2 units by 2 units

Max Area

$2 \times 2 = 4 \text{ units}^2$

2. The sum of two positive numbers is 96. What is the minimum value of the sum of the squares of the two numbers?

Let $x = 1^{st}\ \#$
$y = 2^{nd}\ \#$

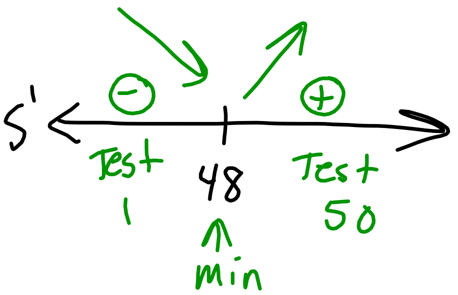
$Sum = x^2 + y^2$

Secondary
$x + y = 96$
$x = \boxed{96 - y}$

$Sum = (96-y)^2 + y^2$
$S = 9216 - 192y + y^2 + y^2$
$S = 2y^2 - 192y + 9216$

$0 < x < 96$
$0 < y < 96$

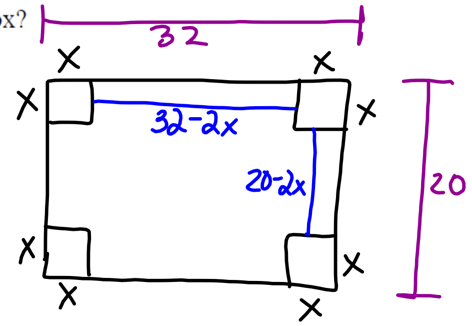
$S' = 4y - 192$
$0 = 4y - 192$
$\frac{192}{4} = \frac{4y}{4}$
$48 = y$



$48 = y$
Find x:
$x + y = 96$
$x + 48 = 96$
$x = 48$

Minimum Sum of squares $= 48^2 + 48^2$
$= \boxed{4608}$

3. A rectangular box, open on the top, is to be constructed from a 20in. by 32in. piece of sheet metal by cutting identical squares from each of the corners and folding up the flaps. What is the length of the sides of the squares that will maximize the volume of the box? What is the maximum volume of the box?



Max Volume: LWH

$V=(32-2x)(20-2x)(x)$

$V=(640-64x-40x+4x^2)(x)$

$V=4x^3-104x^2+640x$

$V'=12x^2-208x+640$

$0=4(3x^2-52x+160)$

$0=3x^2-52x+160$

$3x^2-40x \mid -12x+160$

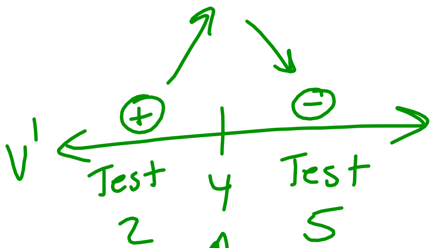
$x(3x-40) \mid -4(3x-40)$

$(x-4)(3x-40)$

$x=4$ $\quad x=\frac{40}{3}$ reject

Rest:

$0<x<10$



Max @ $x=4$

Sides of each square 4in

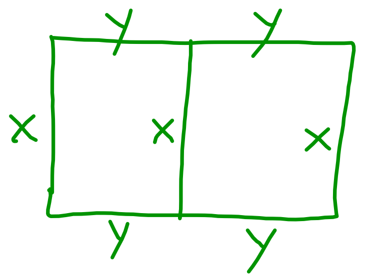
Max Volume:

$V=(32-2(4))(20-2(4))(4)$

$=(24)(12)(4)$

$=1152 \text{ inches}^3$

4. A farmer needs to fence in a rectangular plot of land and then divide it equally using a section of fence running parallel to two sides of the plot. What is the minimum length of fence needed to enclose an area of 6144 square feet?



Minimum Perimeter $= 4y + 3x$

$P = 4\left(\frac{3072}{x}\right) + 3x$

$P = 12288x^{-1} + 3x$

$P' = -12288x^{-2} + 3$

Secondary

$2yx = 6144$

Test:

$x > 0$

$y > 0$

$\frac{6144}{2x} = y$

$\frac{3072}{x} = y$

$x^2\left(0 = -\frac{12288}{x^2} + 3\right)$

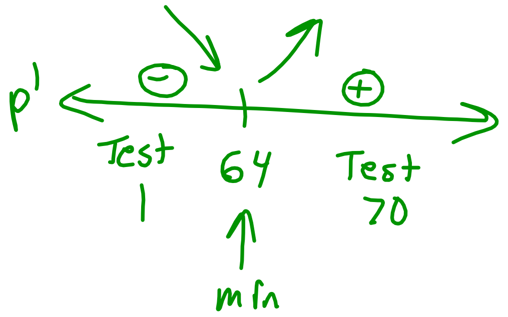
$0 = -12288 + 3x^2$

$\frac{12288}{3} = \frac{3x^2}{3}$

$4096 = x^2$

$\pm 64 = x$

$64 = x$



$x = 64$

Find y:

$2y(64) = 6144$

$\frac{128y}{128} = \frac{6144}{128}$

$y = 48$

Minimum Fence:

$4(48) + 3(64) = \boxed{384 \text{ ft}}$

5. Green Giant needs to make a can that can hold a volume of 64π cubic inches. What are the dimensions of the can that will use the minimum material? What is that amount of material?

Min Surface Area $= 2\pi r^2 + 2\pi r h$

Secondary:
$V = \pi r^2 h$

$\frac{64\pi}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$

$\frac{64}{r^2} = h$

Rest:
$r > 0$
$h > 0$

$SA = 2\pi r^2 + 2\pi \cancel{r}\left(\frac{64}{r^{\cancel{2}}}\right)$

$SA = 2\pi r^2 + 128\pi r^{-1}$

$SA' = 4\pi r - 128\pi r^{-2}$

$r^2\left(0 = 4\pi r - \frac{128\pi}{r^2}\right)$

$0 = 4\pi r^3 - 128\pi$

$\frac{128\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$

$32 = r^3$

$\sqrt[3]{32} = r$

$3.17 \approx r$

SA': $\ominus$ Test 1 | $\sqrt[3]{32}$ | $\oplus$ Test 4

min

$\boxed{r = \sqrt[3]{32} \text{ in}}$

$\boxed{h = \frac{64}{(\sqrt[3]{32})^2} \text{ in}}$

Min SA:

$SA = 2\pi\left(\sqrt[3]{32}\right)^2 + 2\pi\left(\sqrt[3]{32}\right)\left(\frac{64}{(\sqrt[3]{32})^2}\right)$

$\approx \boxed{189.99 \text{ in}^2}$

6. A drilling company has determined that the cost per hour to operate a drill is given by $C(x) = -40x + x^2 + 520$ where x is the speed of the drill. At what speed will the cost per hour be a minimum? What is the hourly cost?

Minimum Cost: $C(x) = -40x + x^2 + 520$

Restrictions:

$x \geq 0$

$C(0) = -40(0) + 0^2 + 520$

$C(0) = 520$

$C' = -40 + 2x$

$0 = -40 + 2x$

$40 = 2x$

$20 = x$

C': $\ominus$ | $\oplus$

Test 0   20   Test 25

$\boxed{\text{Speed} = 20 \text{ units/hr}}$

$C(20) = -40(20) + 20^2 + 520$

$= -800 + 400 + 520$

$C(20) = 120$

$\boxed{\text{Cost} = \$120/\text{hr}}$

#2 from in class

$90 > x > 0 \quad 90 > y > 0$

Sum $= 90$

$x =$ 1st #

$y =$ 2nd #

Max product: $x \cdot y^2$

$$x + y = 90$$
$$-y \quad -y$$
$$x = 90 - y$$

$$P = (90 - y)(y^2)$$
$$P = 90y^2 - y^3$$
$$P' = 180y - 3y^2$$
$$0 = 180y - 3y^2$$
$$0 = 3y(60 - y)$$

$y = 0$ reject | $y = 60$

P': $+$ | 60 | $-$

Test 1 | Test 61

max

$y = 60$

$x = 90 - 60 = 30$

B) $\boxed{60}$